Sample Problems for University of Idaho Math Contest, Grade 12 division

(1) (2020 AMC 12A, Problem 4) How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) have only even digits and are divisible by 5?

(2) (2020 AMC 12A, Problem 5) The 25 integers from -10 to 14, inclusive, can be arranged to form a 5×5 square in which the sum of the numbers in row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

(3) (2020 AMC 12A, Problem 9) How many solutions does the equation $\tan(2x) = \cos(\frac{x}{2})$ have on the interval $[0, 2\pi]$? (Recall that $\sin(2x) = 2\sin x \cos x$, $\cos(2x) = \cos^2 x - \sin^2 x$, and $\cos(\frac{x}{2}) = \pm\sqrt{\frac{1+\cos x}{2}}$.)

(4) (2020 AMC 12A, Problem 13) There are 3 integers a, b, and c, each greater than 1, such that

$$\sqrt[a]{N\sqrt[b]{N\sqrt[c]{N}}} = \sqrt[36]{N^25}$$

for all $N > 1$. What is b?

(5) (2020 AMC 12A, Problem 15) In the complex plane, let A be the set of solutions to $z^3 - 8 = 0$ and let B be the set of solutions to $z^3 - 8z^2 - 8z + 64 = 0$. What is the maximum distance between a point of A and a point of B?

(6) (2020 AMC 12A, Problem 17) The vertices of a quadrilateral lie on the graph of $y = \ln x$, and the x coordinates of these vertices are consecutive positive integers. The area of the quadrilateral is $\ln(\frac{91}{90})$. What are the x-coordinates of the vertices?

(7) (2020 AMC 12A, Problem 19) There exists an increasing sequence of nonnnnegative integers $a_1 < a_2 < \cdots < a_k$ such that

$$\frac{2^{289}+1}{2^{17}+1} = 2^{a_1} + 2^{a_2} + \cdots + 2^{a_k}.$$

What is k?

(8) (2020 AMC 12A, Problem 24) Suppose the triangle ABC is an equilateral triangle of side length s, with the property that there is a point P inside the triangle such that $AP = 1$, $BP = \sqrt{3}$, and $CP = 2$. What is s?

(9) (2020 AIME, Problem 2) There is a unique number x such that the three numbers $\log_8(2x)$, $\log_4 x$, and $\log_2 x$, in that order, form a geometric progression with a positive common ratio. What is that number?

(10) (2020 AIME, Problem 3) A positive integer N is written as abc in base-eleven and $1bca$ in base-eight, where a, b, and c are (not necessarily different from each other) digits. Find all such numbers (and give the answer in base ten).