# Sample Problems for University of Idaho Math Contest, Grade 10 division

(1) (2020 AMC 10A, Problem 2) The numbers 3, 5, 7, $a$, and $b$ have an average (arithmetic mean) of 15. What is the average of $a$ and $b$?

(2) (2020 AMC 10A, Problem 3) Assuming $a \neq 3$, $b \neq 4$, and $c \neq 5$, simplify

$$\left(\frac{a-3}{5-c}\right)\left(\frac{b-4}{3-a}\right)\left(\frac{c-5}{4-b}\right)$$

as much as possible.

(3) (2020 AMC 10A, Problem 6) How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) have only even digits and are divisible by 5?

(4) (2020 AMC 10A, Problem 7) The 25 integers from $-10$ to 14, inclusive, can be arranged to form a $5 \times 5$ square in which the sum of the numbers in row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

(5) (2020 AMC 10A, Problem 12) Triangle $AMC$ is isoceles with $AM = AC$. Medians $\overline{MV}$ and $\overline{CU}$ are perpendicular to each other, and $MV = CU = 12$. What is the area of the triangle $AMC$?

(6) (2020 AMC 10A, Problem 17) Define

$$P(x) = (x - 1^2)(x - 2^2) \cdots (x - 100^2)$$

How many integers $n$ are there such that $P(n) \leq 0$?

(7) (2020 AMC 10A, Problem 20) Quadrilateral $ABCD$ satisfies$\angle ABC = \angle ACD = 90°$, $AC = 20$, and $CD = 30$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point $E$, and $AE = 5$. What is the area of quadrilateral $ABCD$?

(8) (2020 AMC 10A, Problem 22) For how many positive integers $n \leq 1000$ is

$$\lfloor \frac{998}{n} \rfloor + \lfloor \frac{999}{n} \rfloor + \lfloor \frac{1000}{n} \rfloor$$

not divisible by 3? (Note: The notation $\lfloor x \rfloor$ means the greatest integer less than or equal to $x$, or $x$ rounded down.)

(9) (2020 AMC 10A, Problem 25) Jason rolls three fair standard six-sided dice. Then he looks at the rolls and chooses a subset of the dice (possibly empty, possibly all three dice) to reroll. After rerolling, he wins if and only if the sum of the numbers face up on the three dice is exactly 7. What should Jason's strategy be?

(10) (2020 AIME, Problem 3) A positive integer $N$ is written as $abc$ in base-eleven and $1bca$ in base-eight, where $a$, $b$, and $c$ are (not necessarily different from each other) digits. Find all such numbers (and give the answer in base ten).