University of Idaho
High School Mathematics Competition 2023

Competition Guidelines

- Students may only use paper and a writing implement. Other aids (such as calculators, phones, laptops, and sliderules) are not allowed.
- The competition consists of 8 questions. Teams will have 90 minutes.
- Each problem will be graded out of 10 points, for a maximum total of 100 points.
- Every solution should contain a succinct explanation of why the answer is correct or how the team arrived at the answer. Even for a correct answer, this explanation will be taken into consideration in the grading.
- Write your team number in the space provided on each problem. (The problems will be split up for grading.) Please DO NOT write any other identifying information (such as your name or school) on any pages turned in.
- If you require additional pages to write your solution to a problem, use blank paper and staple it to the problem page.
- In the highly unlikely event of a tie score, the team that submitted its solutions earlier will be deemed the winner (of the tie).

University of Idaho
High School Mathematics Competition 2023

Team Number: ☐

Division I, Problem 1

Alice can paint a wall by herself in one hour. Bob needs 90 minutes to paint the same wall by himself. If Alice and Bob work together, how long will it take for them to paint that wall? What fraction of the wall will Alice paint? Assume that they don't distract each other and each paints at the same rate they would if they were painting by themselves.

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division I, Problem 2

Find all ways that the number 500 can be expressed as the sum of some number of consecutive positive integers. (For example, $39 = 19 + 20$ and $39 = 12 + 13 + 14$ are two ways of writing 39 as the sum of consecutive integers.)

University of Idaho
High School Mathematics Competition 2023

Team Number: ☐

Division I, Problem 3

Consider the parabola (given by an equation $y = ax^2 + bx + c$ for some numbers a, b, and c which we are not telling you) that goes through the points $(2023, -5)$, $(2035, -5)$, and $(2029, 4)$. Where does this parabola cross the x-axis?

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division I, Problem 4

An isoceles triangle with side lengths 2, 4, and 4 is inscribed in a circle. Find the radius of the circle.

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division I, Problem 5

A sequence of numbers is constructed as follows. The first number is 2023^{2023}. If a number n in the sequence is a perfect square, then the next number in the sequence is $\sqrt{n}$. Otherwise, the next number is $n+1$. What is the smallest number in this sequence (which may happen more than once)?

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division I, Problem 6

Find two rectangles R and S, both with integer side lengths, so that the perimeter of R is twice the perimeter of S and the area of S is twice the area of R.

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division I, Problem 7

What is the highest power of 2 that divides $101 \times 102 \times 103 \times \cdots \times 200$?

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division I, Problem 8

Yana and Zoe have a jar with 2023 marbles, which they use to play a game. Each player, on her turn, gets to take at least 1 and at most 23 marbles out of the jar. The player who takes the last marble wins. (For example, if there are 17 marbles left in the jar, the player whose turn it is can win by taking all the marbles.) Suppose Yana goes first. Describe a strategy that Yana can use to make sure she wins the game no matter what Zoe does. In particular, how many marbles should Yana take on her first turn?

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 1

Alice can paint a wall by herself in 80 minutes. Bob needs 3 hours to paint the same wall by himself. If Alice and Bob work together, how long will it take for them to paint that wall? What fraction of the wall will Alice paint? Assume that they don't distract each other and each paints at the same rate they would if they were painting by themselves.

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 2

Find a positive integer $n < 100$ so that $\frac{n^6}{2^n} \leq 10^{-6}$, together with an explanation by hand that your answer satisfies those conditions.

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 3

How many numbers x, with $0 \leq x \leq \pi/2$ are there so that

$$\frac{2n+2}{n^2+1} \sin x$$

is an integer for some integer n?

University of Idaho
High School Mathematics Competition 2023

Team Number: ☐

Division II, Problem 4

Suppose we have a regular triangular pyramid (one where every face is an equilateral triangle of the same size) of volume $\frac{\sqrt{2}}{12}$. Construct a smaller regular triangular pyramid by using the centroids of the 4 faces as the vertices. What is the volume of the smaller pyramid?

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 5

For the purposes of this problem, we say that a number n is **beautiful** if $2n$ is a perfect square and $3n$ is a perfect cube. Find all beautiful numbers less than 10000. (Hint: There are two of them, but you have to justify this fact.)

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 6

Solve the equation
$$x^2 + \frac{1}{x^2} - 5 - \frac{5}{x} + 6 = 0.$$

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 7

You are told that the polynomial $2x^4 + ax^2 + b$ is divisible by the polynomial $x^2 - 2x + 3$. Find a and b.

University of Idaho
High School Mathematics Competition 2023

Team Number:

Division II, Problem 8

Yana and Zoe have a jar with 2023 marbles, which they use to play a game. Each player, on her turn, gets to take at least 1 and at most 23 marbles out of the jar. The player who takes the last marble wins. (For example, if there are 17 marbles left in the jar, the player whose turn it is can win by taking all the coins.) Suppose Yana goes first. Describe a strategy that Yana can use to make sure she wins the game no matter what Zoe does. In particular, how many marbles should Yana take on her first turn?